Planar Topology: Discrete Sets

Discreteness and Compact Subsets

Lecture Notes

October 15, 2026

1 Discrete Sets

Definition (discrete set)

Let U be open. $I \subseteq U$ is **discrete in** U if it has no limit points in U, i.e.

$$\forall z \in U \ \exists \delta > 0 : \big(B(z,\delta) \setminus \{z\}\big) \cap I = \varnothing.$$

Example. $z_n = 1/n$ is *not* discrete in $\mathbb{C}$ (it has limit point 0), but *is* discrete in $\mathbb{C} \setminus \{0\}$.

Lemma

Let $K \subseteq U$ be closed and bounded, and I discrete in U. Then $K \cap I$ is finite.

Proof. If not, there is an infinite sequence of distinct points $(z_k) \subseteq I \cap K$. Since (z_k) is bounded, it has a limit point z; since K is closed and $\{z_k\} \subseteq K$, $z \in K \subseteq U$. But then z is a limit point of I lying in U, contradicting that I is discrete in U. $\square$